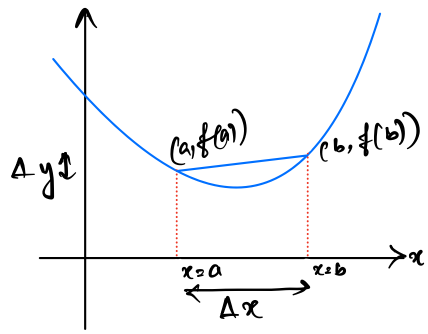
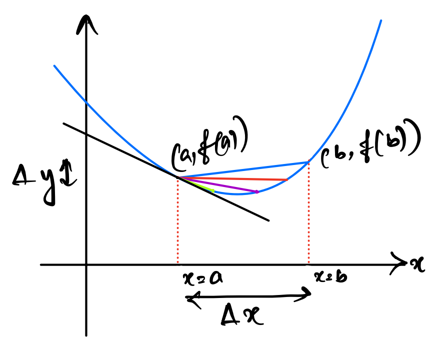
Ch.3 Defining the Derivative:

To find the average rate of change between two distinct points, we find the slope of the secant line.



$$\text{slope (secant)} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(b) - f(a)}{b - a}$$

If we want to find the rate of change at one single point, then we move the other point along the curve towards the required point.



$$\text{slope (tangent)} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{b \to a} \frac{f(b) - f(a)}{b - a}$$

Def^n: Let $f(x)$ be defined in an interval containg a. Then the derivative (or the slope of the tangent) at $x = a$ is given by

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\text{or} \quad \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

} Instantaneous rate of change relative to x.

Eg. $f(x) = x^2$, Find $f'(x)$ at $x = 3$ & then find the equation of the tangent to $f(x)$ at $x=3$.

$\rightarrow$ $f(x) = x^2$

Then $f'(a) = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

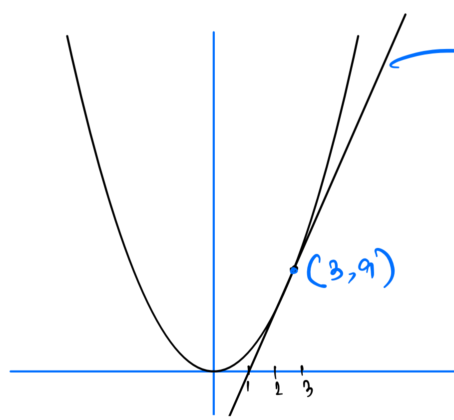
Replace a by 3, so

$$f'(3) = \lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \to 3} \frac{x^2 - 3^2}{x - 3}$$

$$= \lim_{x \to 3} \frac{(x+3)(x-3)}{(x-3)} \quad [\text{Note: } x-3 \neq 0]$$

$$= \lim_{x \to 3} (x+3) = 3+3 = 6$$



tangent line with slope at $x=3$ is 6

In this example, tangent line is $y = 6x + b$, where the tangent passes through $(3, 9)$ point.

So, $9 = 6(3) + b \Rightarrow b = -9$

So, tangent line is, $y = 6x - 9$

Let $f(x) = e^x$. Find $f'(x)$, if $\lim\limits_{x \to 0} \frac{e^x - 1}{x} = 1$.

$\rightarrow$ $f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$= \lim_{h \to 0} \frac{e^x [e^h - 1]}{h}$$

$$= e^x \cdot \lim_{h \to 0} \frac{e^h - 1}{h}$$

$$= e^x \cdot 1 = e^x$$

(*) Given, $\lim\limits_{h \to 0} \frac{e^h - 1}{h} = 1$